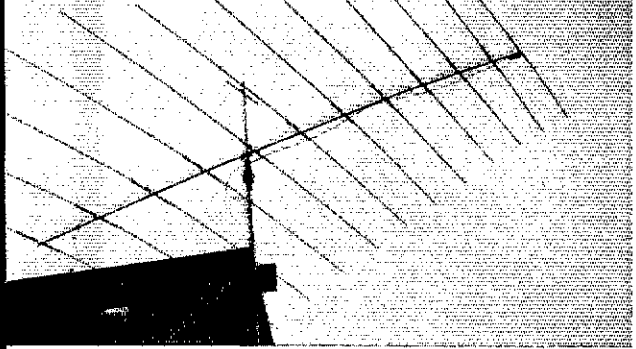

The Log-Periodic Dipole Array

Theory, Design, and Construction of a Practical Antenna for HF Work

BY PETER D. RHODES,* K4EWG

THIS ARTICLE is written to familiarize the amateur with the log-periodic dipole array (LPDA), and to provide the basic theory, design procedures and the construction of a practical antenna such as that used at the author's QTH. In the discussion the mathematical derivation of individual element currents, voltages, and admittances has been omitted for simplicity. The amateur with a solid background in differential calculus, vector algebra and simultaneous differential-equation matricies can pursue this area using the reference material. However, derivation of the mathematical model for these parameters is not necessary in a practical design consideration.

The LPDA has had relatively little use in amateur applications and has been presented sparingly in vhf and uhf articles; however, it will be seen that a good LPDA for any band, hf to uhf, can be built to meet the amateur's requirements at nominal cost: high forward gain, good front-to-back ratio, low VSWR, and a boom length equivalent to a full sized three-element Yagi.

The LPDA is a frequency-independent antenna invented by DuHamel and Isbell.[1] It is in wide use by the armed forces. The LPDA exhibits a relatively low SWR (usually not greater than 2 to 1) over a wide band of frequencies. Carrel[2] has shown that a well designed LPDA can yield a 1.3 to 1 SWR over a 1.8 to 1 frequency range with a directivity of 9.5 dB.†

Basic Theory

The LPDA is frequency independent in that the electrical properties such as the mean resistance level, Ro, characteristic impedance of the feed line Zo, and driving-point admittances, Yo, vary periodically with the logarithm of the frequency. See Fig. 1. As the frequency f_1 is shifted to another frequency f_2 within the passband of the antenna, the relationship is $f_2 = f_1/\tau$

where τ = a design parameter, a constant; $\tau < 1.0$.

Also $f_3 = f_1/\tau^2$

$f_4 = f_1/\tau^3$

$\cdot$

$\cdot$

$\cdot$

$$f_n = f_1/\tau^{n-1} \qquad \text{(Eq. 1)}$$

where $n = 1, 2, 3, \ldots, n$

f_1 = lowest frequency

f_n = highest frequency

Taking the log of Eq. 1,

$$\log f_n = \log f_1 - (n-1)\log\tau \qquad \text{(Eq. 2)}$$

Eq. 2 shows that any property shown on a log f scale is periodic with period log τ.

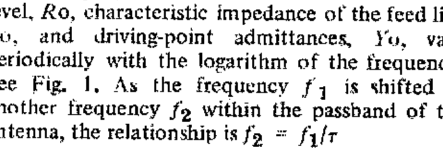

Fig. 1 — Showing the periodic variations of an electrical property of the array versus the logarithm of the frequency.

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[1] For this and subsequent references, refer to the bibliography at the end of this article.

†[EDITOR'S NOTE: Directivity is the ratio of maximum radiation intensity in the forward direction to the average radiation intensity from the array. Assuming no resistive losses in the antenna system, 9.5 dB directivity equates to 9.5 dB gain over an isotropic radiator or approximately 7.4 dB gain over a half-wave dipole.]

The design parameter τ is a geometric constant near 1.0 which is used to determine the element lengths, l, and element spacings, d. See Fig. 2. That is,

$$l_2 = \tau l_1$$
$$l_3 = \tau l_2$$
$$\vdots$$
$$l_n = \tau l_{(n-1)} \qquad \text{(Eq. 3)}$$

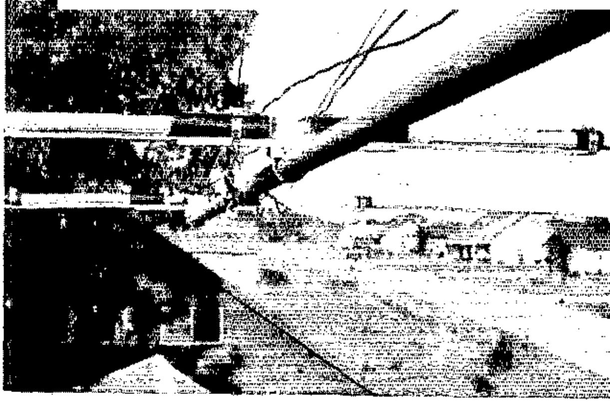
where l_n = shortest element length, and

$$d_{23} = \tau d_{12}$$
$$d_{34} = \tau d_{23}$$
$$\vdots$$
$$d_{n-1,n} = \tau d_{n-2,n-1} \qquad \text{(Eq. 4)}$$

where d_{23} = spacing between elements 2 and 3.

Each element is driven with a phase shift of 180° by switching or alternating element connections, as shown in Fig. 2. The dipoles near the input, being nearly out of phase and close together, nearly cancel each others' radiation. As the element spacing, d, expands there comes a point along the array where the phase delay in the transmission line combined with the 180° switch gives a total of $360^\circ (1 - d/\lambda)$. This puts the radiated fields from the two dipoles d apart in phase in a direction toward the apex. Hence, a lobe coming off the apex results when the total phase delay from one dipole to the next is $360^\circ (1 - d/\lambda)$.[5]

This phase relationship exists in a set of dipoles known as the "active region." If we assume that an LPDA is designed for a given frequency range, then that design must include an active region of dipoles for the highest and lowest design frequency. It has a bandwidth which we shall call *Bar* (bandwidth of the active region).[2] Cheong,[5] using a high speed computer, has made an extensive study of a 12-element LPDA. He determined the individual element currents, both real and imaginary. The dipole nearest resonance is his element number 6. The imaginary parts of the currents in shorter elements 7 to 12 are capacitive, while those in longer elements 1 to 6 are inductive. The capacitive current components in shorter elements 9 and 10 exceed the conductive components; hence, these elements receive little power from the feeder and act as parasitic directors. The inductive current components in longer elements 4 and 5 are dominant and they act like parasitic reflectors. Elements 6, 7, and 8 receive most of their power from the feeder and act like driven elements. The amplitudes of the currents in the remaining elements are small and they may be ignored as primary contributors to the radiation field. Hence, we have a generalized Yagi array with seven elements comprising the active region. It should be noted that this active region is for a specific set of design parameters (τ = 0.93, σ = 0.175). The number of elements making up the active region will vary with τ and σ. Adding additional elements *on either side of the active region cannot significantly* modify the circuit or field properties of the array.[4]



This close-up view shows the element-to-boom, mounting arrangement. Two hose clamps secure each half-element to the angle-aluminum and aluminum-bar supports. Above the nearer element the feeder conductors and the strut cables are visible.

This active region determines the basic design parameters for the array, and sets the bandwidth for the structure, *Bs*. That is, for a design-frequency coverage of bandwidth B, there exists an associated bandwidth of the active region such that

$$B_s = B \times Bar \qquad \text{(Eq. 5)}$$

where B = operating bandwidth $= \dfrac{f_n}{f_1}$ (Eq. 6)

and f_1 = lowest freq., MHz

f_n = highest freq., MHz

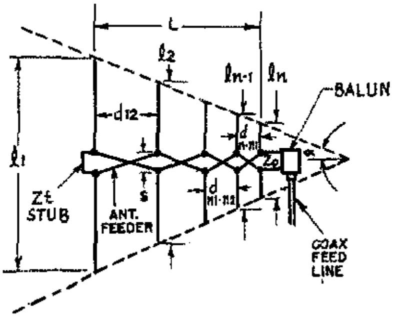


Fig. 2 — The log-periodic dipole array and some of the design parameters.

$$\tau = \frac{l_n}{l_{n-1}} = \frac{d_{n,n-1}}{d_{n-2,n-1}}$$

$$\sigma = \frac{d_{n,n-1}}{2l_{n-1}}$$

$$h_n = \frac{l_n}{2}$$

Where l = el. length
h = el. half length
d = el. spacing
τ = design constant
σ = relative spacing constant
S = feeder spacing
Zo = char. impedance of antenna feeder

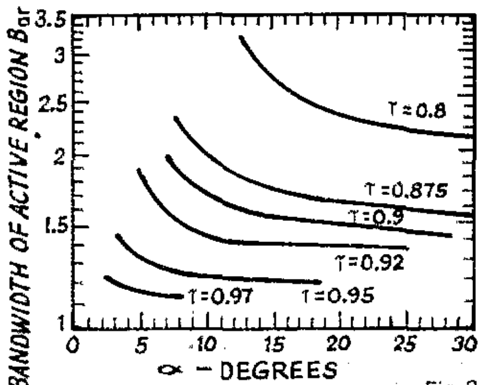


Fig. 3

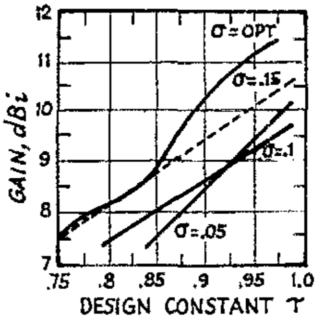


Fig. 4

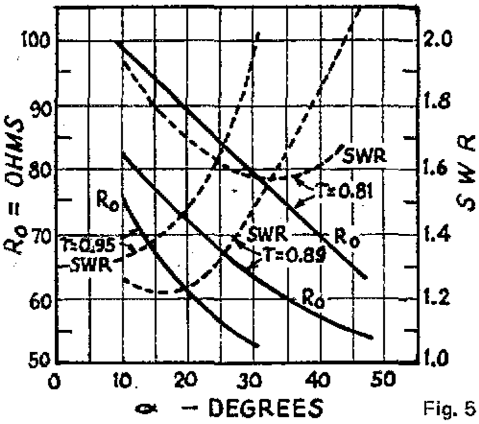


Fig. 5

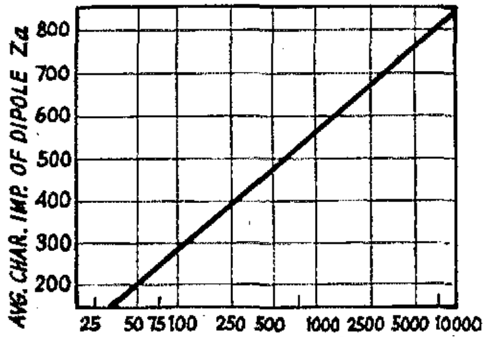


Fig. 6

*B*ar varies with τ and α as shown in Fig. 3. Element lengths which fall outside *B*ar play an insignificant role in the operation of the array. The *gain of an LPDA is determined by the design* parameter τ and the relative element spacing constant σ. There exists an optimum value for σ, σ_{opt}, for each τ in the range $0.8 \leqslant \tau < 1.0$, for which the gain is maximum; however, the increase in gain achieved by using σ_{opt} and τ near 1.0 (i.e., $\tau = 0.98$) is only 3 dB above isotropic (3 dB*i*) when compared with the minimum σ (σ_{min} = .05) and $\tau = 0.9$, shown in Fig. 4.

An increase in τ means more elements and optimum σ means a long boom. In the construction portion of this article we shall see that a well-constructed, high gain (8.5 dB*i*) LPDA can be designed in the hf region with $\tau = 0.9$ and $\sigma = .05$. The relationship of τ, σ, and α is as follows:

$$\sigma = (1/4)(1-\tau)\cot\alpha \qquad \text{(Eq. 7)}$$

where α = 1/2 the apex angle
τ = design constant
σ = relative spacing constant

$$\text{also } \sigma = \frac{d_{n,\,n-1}}{2\,l_{n-1}} \qquad \text{(Eq. 8)}$$

$$\sigma_{opt} = 0.258\,\tau - .066 \qquad \text{(Eq. 9)}$$

The method of feeding the antenna is rather simple. As shown in Fig. 2, a balanced feeder is required for each element, and all adjacent elements are fed with a 180° phase shift by alternating element connections. In this article the term *antenna feeder* is defined as that line which *connects each adjacent element. The feed line is* that line between antenna and transmitter or Transmatch. The characteristic impedance of the antenna feeder, *Zo*, must be determined so that the feed-line impedance and type of balun can be determined. The antenna-feeder impedance *Zo* depends on the mean radiation resistance level *Ro* (required input impedance of the active region elements – see Fig. 5) and average characteristic impedance of a dipole *Za*. (*Za* is a function of element radius *a* and the resonant element half length, where $h = \lambda/4$. See Fig. 6.) The relationship is as follows:

$$Zo = \frac{Ro^2}{8\sigma' Za} + Ro\sqrt{\left(\frac{Ro}{8\sigma' Za}\right)^2 + 1} \qquad \text{(Eq. 10)}$$

where *Zo* = characteristic impedance of feeder
Ro = mean radiation resistance level or required input impedance of the active region.
Za = average characteristic impedance of a dipole

$$= 120\left(\ln \frac{h}{a} - 2.55\right) \qquad \text{(Eq. 11)}$$

h = el. half length
a = radius of el.

$$\sigma' = \text{mean spacing factor} = \frac{\sigma}{\sqrt{\tau}} \qquad \text{(Eq. 12)}$$

Table I – Array dimensions, feet

El. No.	l_n	h	$d_{n-1,n}$ *(spacing)*	*nearest resonant*
1	38.0	19	0	
2	34.2	17.1	3.862 = d_{12}	14 MHz
3	30.78	15.39	3.475 = d_{23}	
4	27.7	13.85	3.13 ·	
5	24.93	12.465	2.815 ·	
6	22.44	11.22	2.533 ·	21 MHz
7	20.195	10.098	2.28 ·	
8	18.175	9.088	2.05 ·	
9	16.357	8.179	1.85 ·	28 MHz
10	14.72	7.36	1.663 ·	
11	13.25	6.625	1.496 ·	
12	11.924	5.962	1.347 = $d_{11,12}$	

From Fig. 5 we can see that Ro decreases with increasing τ and increasing α. Also the VSWR with respect to Ro has a minimum value of about 1.1 to 1 at σ optimum, and a value of 1.8 to 1 at $\sigma = .05$. These SWR values are acceptable when using standard RG-8/U 52-ohm and RG-11/U 72-ohm coax for the feed line. However, a one-to-one VSWR match can be obtained at the transmitter end using a coax-to-coax Transmatch.[6] A Transmatch is used at the author's QTH so that the transmitter low pass filter will see a 52-ohm load on each frequency within the array passband. The Transmatch also eliminates possible harmonic radiation caused by the frequency-independent nature of the array.

Once the value of Zo has been determined for each band within the array passband, the balun and feed line may be chosen. That is, if Zo = 100 ohms, a good choice for the balun would be 1 to 1 balanced to unbalanced, and 72-ohm coax feed line. If Zo = 220 ohms, choose a 4 to 1 balun, and 52-ohm coax feed line, and so on. The balun may be omitted if the array is to be fed with an open-wire feed line.

The *terminating impedance, Zt, may be* omitted. However, if it is used, it should have a length no longer than $\lambda_{max}/8$. The terminating impedance tends to increase the front-to-back ratio for the lowest frequency used and in the construction details a 6-inch shorting jumper wire is shown for Zt. When Zt is simply a short-circuit jumper the longest element behaves as a passive reflector. It also might be noted that one could increase the front-to-back ratio on the lowest frequency by moving the passive reflector (No. 1 element) a distance of 0.15 to 0.25 λ behind element No. 2, as would be done in the case of an ordinary Yagi parasitic reflector. This of course would necessitate lengthening the boom.

As noted in Fig. 7, the front-to-back ratio increases as the frequency increases. This is because more of the shorter inside elements form the active region, and the longer elements become additional reflectors.

Design Procedure

A systematic step-by-step design procedure of the LPDA is to follow. This procedure will provide the amateur with the basic tools for designing any LPDA for any desired bandwidth.

1) Decide on an operating bandwidth B between f_1, lowest frequency and f_n, highest *frequency, using Eq. 6.*

2) Choose τ and σ to give a desired gain (Fig. 4).

$$0.8 \leqslant \tau \leqslant 0.98$$
$$.05 \leqslant \sigma \leqslant \sigma_{opt}$$

The value of σ_{opt} may be determined from Eq. 9.

3) Determine the apex half-angle α

$$\cot \alpha = \frac{4\sigma}{1-\tau}$$

4) Determine the bandwidth of the active group Bar from Fig. 3.

5) *Determine the structure (array) bandwidth* Bs from Eq. 5.

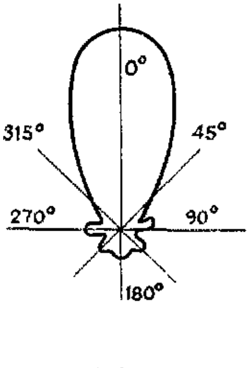


Fig. 7 — Measured radiation pattern for the lowest frequency band of the author's array, 14 MHz. The front-to-back ratio increases as the frequency increases. For this array it is 14.4 dB at 14 MHz, 19.5 dB at 21 MHz, and 21 dB at 28 MHz.

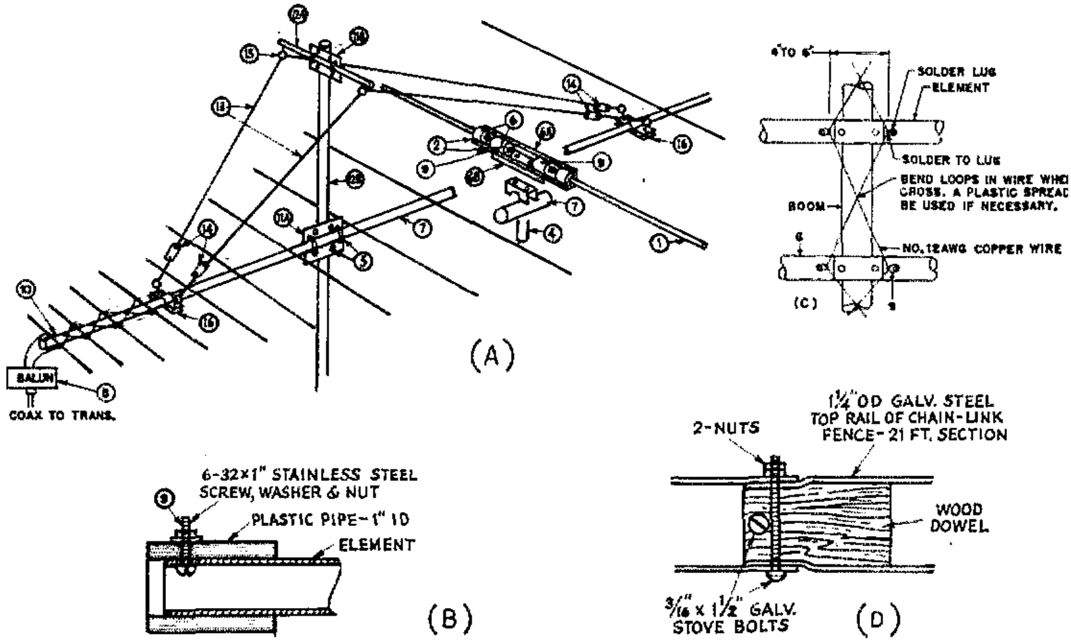


Fig. 8 — Construction diagram. At B and C are shown the method of making electrical connection to each half element, and at D is shown how the boom sections are joined.

6) Determine the boom length, L, number of elements N, and longest element length, l_1.

$$L = \left[\frac{1}{4}\left(1 - \frac{1}{Bs}\right)\cot \alpha\right] \lambda_{\max} \qquad \text{(Eq. 13)}$$

$$N = 1 + \frac{\log Bs}{\log\left(\frac{1}{\tau}\right)} \qquad \text{(Eq. 14)}$$

$$l_1 = \frac{492}{f_1}$$

where $\lambda_{\max}$ = longest free-space wavelength = $984/f_1$

Examine L, N and l_1 and determine whether or not the array size is acceptable at your QTH. If the array is too large, increase α by 5° and repeat steps 2 through 6.

7) Determine the terminating stub Zt. (Note: For hf arrays short out the longest element with a 6-inch jumper. For vhf and uhf arrays use:

$$Zt = \lambda_{\max}/8.$$

8) Once the final values of τ and σ are found, the characteristic impedance of the feeder Zo must be determined so the type of balun and feed line can be found. Use Eq. 10. Determine Ro from Fig. 5, Za from Fig. 6 and σ' from Eq. 12. Note: Values for h/a Za, and Zo must be determined for each amateur band within the array passband. Choose the element half-length h nearest $h = \lambda/4$, at the center frequency of each amateur band. Once Zo is found for each band, choose whatever combination of balun and feed line will give the lowest SWR on each band.

9) Solve for the remaining element lengths from Eq. 3.

10) Determine the element spacing d_{12} from

$$d_{12} = 1/2\,(l_1 - l_2)\cot \alpha \qquad \text{(Eq. 15)}$$

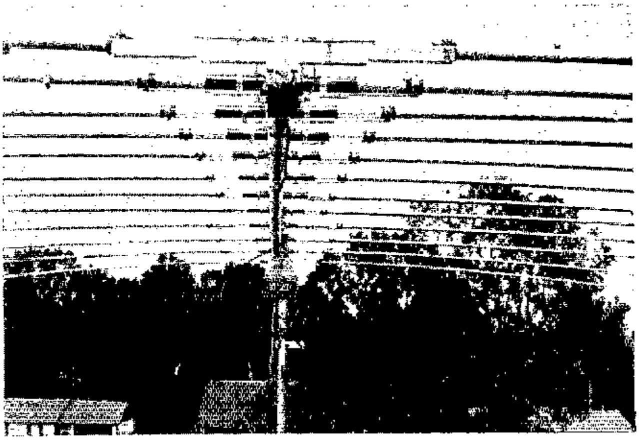
and the remaining element-to-element spacings from Eq. 4. This completes the design.

Construction of an LPDA at the Author's QTH

The final result of this work is the finished project. I wanted one beam antenna which would replace my three stacked monoband 3-element Yagis (10, 15, and 20 meters) and would give comparable performance. The LPDA was chosen, and on-the-air performance substantiates the theory behind it.

The parameters are as follows:

Frequency range, 13-30 MHz
Half-power beamwidth, 43° (14 MHz)
Operating bandwidth, $B = 30/13 = 2.3$
Design parameter $\tau = 0.9$
Relative element spacing constant $\sigma = .05$
Apex half-angle $\alpha = 25°$, $\cot \alpha = 2.0325$
Bandwidth of active group, $Bar = 1.4$
Bandwidth of structure, $Bs = 3.22$
Boom length, $L = 26.5$ ft
Longest element $l_1 = 38$ ft (a tabulation of element lengths and spacings is given in Table I)
Total weight, 116 pounds



Looking "down the throat" of the K4EWG hf LPDA.

Wind-load area, 10.7 sq. ft

Required input impedance (mean resistance), Ro = 67 ohms, Zt = 6-inch jumper No. 18 wire

Average characteristic dipole impedance:. $Za_{14\ MHz} = 450$ ohms; $Za_{21\ MHz} = 420$ ohms; $Za_{28\ MHz} = 360$ ohms.

Mean spacing factor $\sigma' = .0527$.

Impedance of the feeder: $Zo_{14\ MHz} = 95$ ohms; $Zo_{21\ MHz} = 97$ ohms; $Zo_{28\ MHz} = 103$ ohms

Using a toroid balun at the input terminals and a 72-ohm coax feeder the SWR is 1.4 to 1 (maximum).

The mechanical assembly uses materials readily available from most local hardware stores or aluminum supply houses. The materials needed are given in Table II. In the construction diagram, Fig. 8, the materials are referred to by their respective material list number. The photographs show the overall construction picture, and the drawings show the details. Table III gives the required tubing lengths to construct the elements.

The antenna feeder (line connecting adjacent elements) is constructed of No. 12 solid copper wire, with solder-lug connections at each element. The feeder spacing S (Fig. 2) is not critical. From 4 to 6 inches is satisfactory for hf antennas; use smaller spacings for vhf and uhf arrays. For uhf and vhf arrays, S should be small when compared to the smallest wavelength used (i.e., less than $\lambda_{min}/8$.

Results

This beam has been in operation at the author's QTH for several months, and it has performed better than had been anticipated. The front-to-back ratio is approximately 19.5 dB at 21 MHz with a forward gain of 8.5 dB*i*. The radiation pattern, plotted in Fig. 7 shows a good forward lobe at 14 MHz. The beam was well worth the effort, as at a height of only 30 feet it punches through the pileups with moderate power (100 watts).

Table II — Materials list

Material Description	*Quantity*
1. Aluminum tubing — .047" wall thickness	
1" — 12' or 6' lengths	126 lineal feet
7/8" — 12' lengths	96 lineal feet
7/8" — 6' or 12' lengths	66 lineal feet
3/4" — 8' lengths	16 lineal feet
2. Stainless-steel hose clamps — 2" max.	48 ea.
3. Stainless-steel hose clamps — 1-1/4" max.	26 ea.
4. TV-type U-bolts	14 ea.
5. U-bolts, galv. type	
5/16" × 1-1/2"	4 ea.
1/4" × 1"	2 ea.
6. 1" ID polyethelene water-service pipe — 160 psi test, approx. 1-1/4" OD	20 lineal feet
A. 1-1/4" × 1-1/4" × 1/8" aluminum angle — 6' lengths	30 lineal feet
B. 1" × 1/4" aluminum bar — 6' lengths	12 lineal feet
7. 1-1/4" top rail of chain-link fence	26.5 lineal feet
8. 1:1 toroid balun	1 ea.
9. 6 — 32 × 1" stainless-steel screws	24 ea.
6 — 32 stainless-steel nuts	48 ea.
No. 6 solder lugs	24 ea.
10. No. 12 copper feeder wire	60 lineal feet
11.	
A. 12" × 8" × 1/4" aluminum plate	1 ea.
B. 6" × 4" × 1/4" alum. plate	1 ea.
12.	
A. 3/4" galv. pipe	3 lineal feet
B. 1" galv. pipe — mast	5 lineal feet
13. Galv. guy wire	50 lineal feet
14. 1/4" × 2" turnbuckles	4 ea.
15. 1/4" × 1-1/2" eye bolts	2 ea.
16. TV guy clamps and eye bolts	2 ea.

Table III – Element material requirements

El. No.	*1" tubing*		*7/8" tubing*		*3/4" tubing*		*1-1/4" angle*	*1" bar*
	Lth.	*Qty.*	*Lth.*	*Qty.*	*Lth.*	*Qty.*	*Lth.*	*Lth.*
1	6'	2	6'	2	8'	2	3'	1'
2	6'	2	12'	2	—	—	3'	1'
3	6'	2	12'	2	—	—	3'	1'
4	6'	2	8.5'	2	—	—	3'	1'
5	6'	2	7'	2	—	—	3'	1'
6	6'	2	6'	2	—	—	3'	1'
7	6'	2	5'	2	—	—	2'	1'
8	6'	2	3.5'	2	—	—	2'	1'
9	6'	2	2.5'	2	—	—	2'	1'
10	3'	2	5'	2	—	—	2'	1'
11	3'	2	4'	2	—	—	2'	1'
12	3'	2	4'	2	—	—	2'	1'

This article has dealt with the basic LPDA system. However, there are endless high-gain array possibilities with this type of antenna. Tilting the elements toward the apex will increase the gain 3 to 5 dB. Adding parasitic directors and a reflector will increase both gain and front-to-back ratio for a specific frequency within the passband. The LPDA-Yagi combination is very simple. Use the LPDA design procedures within the set of driven elements, and place parasitic elements at normal Yagi spacings from the LPDA end elements. Use standard Yagi design procedures for the parasitic elements. An example of a single band high-gain LPDA-Yagi would be a two- or three-element LPDA for 21.0 to 21.45 MHz with the addition of 2 or 3 parasitic directors and one parasitic reflector. The combinations are endless.

I wish to thank Ben Painter, W4BBP, who helped in the plotting of the radiation pattern. I also wish to thank the fellows at W3SK, who helped in on-the-air far-field testing.

References

1) Isbell, "Log Periodic Dipole Arrays," *IRE Transactions on Antennas and Propagation*, Vol. AP-8, No. 3, May, 1960, pp. 260-267.

2) Carrel, "The Design of Log-Periodic Dipole Antennas," *1961 IRE International Convention Record*, Part 1, Antennas and Propagation, pp. 61-75; also Ph. D. thesis, University of Ill., Urbana, Ill., 1961.

3) Rumsey, *Frequency Independent Antennas*, pp. 71-78, Academic Press, N.Y., 1966.

4) King, Mack, and Sandler, *Arrays of Cylindrical Dipoles*, pp. 244-269, Cambridge Univ. Press, London, 1968.

5) Cheong, *Arrays of Unequal and Unequally Spaced Dipoles*, Ph. D. Thesis, Harvard Univ., Cambridge, Mass., 1967.

6) McCoy, "The Ultimate Transmatch," *QST*, July, 1970, p. 24.

The Rollerless Ultimate

(Continued from page 12)

small amount of rf energy to the input connector (no more power than necessary to get a reading on the SWR indicator in its most sensitive position) and adjust both the INPUT and OUTPUT controls while observing the directional wattmeter set for reflected power. Various settings of the capacitors should be used with each switch position until the proper settings are established as indicated by minimum reflected power. The operator may then log the settings and return to them anytime.

In some cases it may be found that two positions of the inductor switch may be used, either of which will provide proper reflected power readings. When this situation occurs, the position which allows the output capacitor to be more fully meshed is the one to use. In tests, the author could match any of the station antennas on *any* band, with the exception of ten meters. The lead inductance and stray capacitance of the circuit makes it difficult to load some "odd" antennas on this band. Ordinary antennas, such as a ten-meter Yagi or dipole can be loaded properly, however.

This Transmatch was designed with 160-meter operation in mind. The final product proved to be marginal in performance since there was not sufficient capacitance available at C2 for loading an 80-meter open-wire-fed dipole on 160 meters. Additional capacitance connected in parallel with C2 solves the problem nicely. A transmitting ceramic 5-kV, 100-pF capacitor is satisfactory for this purpose and can be included along with an appropriate switch if 160-meter operation is anticipated. If there is a need to include an SWR meter in the Transmatch, the constructor is referred to the Monimatch circuit published with the Ultimate Transmatch which appears in *QST* for July, 1971, or any recent edition of *The Radio Amateur's Handbook*.